Embedding a forward model of barotropic and baroclinic tides into HYCOM

Brian K. Arbic
Institute for Geophysics
The University of Texas at Austin

Will be working with Eric, Harley, Joe, and Alan to insert tides into HYCOM.

Builds on:


Simmons, Hallberg, Arbic (2004)

Arbic, MacAyeal, Mitrovica, Milne (2004)

Arbic (2005)
Motivation

Models of tides and non-tidal motions currently run separately—why not simultaneously?

Both tides and wind-driven motions provide energy sources for mixing, which affects large scale circulation.

Would like to have model including both, which derives mixing from dissipation, and dissipation from drag acting on both types of motions.

See poster for work on dissipation of eddies in idealized models.

Here describe work done on forward tide modeling, which has recently made great strides.

Tidal forcing periodic in time, has simple structure in space (Legendre polynomials), thus is good test of general circulation models.
Tidal dissipation and forward models

Total dissipation inferred from astronomy $3.7 \, \text{TW} \Rightarrow 10 \, \text{mW m}^{-2}$ areal average.

Models with only quadratic drag put all dissipation into shallow seas:

$$< \rho_0 c_d |\tilde{u}|^3 > =$$

0.02 mW m$^{-2}$, $|\tilde{u}| = 2 \, \text{cm s}^{-1}$,
323 mW m$^{-2}$, $|\tilde{u}| = 50 \, \text{cm s}^{-1}$.

Alternative view: internal wave breaking over rough topography significant energy sink.

Egbert and Ray (2000, 2001): T/P-constrained models yield $\sim 1$ TW dissipation over mid-ocean rough topography, in agreement with in-situ evidence (e.g. Polzin et al. 1997).

Topographic drag schemes

JS: analytical result for wave drag for flow $\bar{u}$ over monochromatic terrain $h \sin(kx)$, buoyancy frequency $N$:

$$\text{drag} = \frac{1}{2}Nkh^2 \bar{u}, \ h=\text{rms residual of unresolved topography}, \ k = \text{tunable parameter}.$$ 

We use scheme of Garner (2005), which builds on analytical result for drag on steady flow over arbitrary topography and includes scalings for nonlinear effects at bottom.
One-layer equations

\[
\frac{\partial \eta}{\partial t} + \nabla \cdot [(H + \eta)\bar{u}] = 0
\]

\[
\frac{\partial \bar{u}}{\partial t} + (f + \zeta)\hat{k} \times \bar{u} = -g\nabla(\eta - \eta_{EQ} - \eta_{SAL})
\]

\[
-\nabla(\frac{1}{2} \bar{u} \cdot \bar{u}) + \nabla \cdot \left[ K_H(H + \eta)\nabla\bar{u} \right] - \frac{c_d|\bar{u}|\bar{u}}{H + \eta} + \frac{T\bar{u}}{\rho_0(H + \eta)}
\]

\( H \): resting thickness

\( \eta \): perturbation surface elevation

\( \bar{u} \): velocity

\( f \): Coriolis parameter

\( \zeta = \hat{k} \cdot (\nabla \times \bar{u}) \)

\( K_H \): horizontal friction

\( c_d \): quadratic drag coefficient

\( T \): topographic drag tensor
Periods and frequencies
\((\omega = 2\pi/\text{period})\) of celestial motions and of tides

Mean solar day: 1 mean solar day, \(\omega_0\)
Mean lunar day: 1.0351 mean solar days, \(\omega_1\)
Sidereal month: 27.3217 mean solar days, \(\omega_2\)
Tropical year: 365.2422 mean solar days, \(\omega_3\)
Moon’s perigee: 8.85 Julian years, \(\omega_4\)
Sidereal day: 0.9973 solar days, \(\omega_s = \omega_0 + \omega_3 = \omega_1 + \omega_2\)

Frequencies of four largest semidurnal tides:
\(M_2: 2\omega_1\)
\(S_2: 2\omega_0\)
\(N_2: 2\omega_1 - \omega_2 + \omega_4\)
\(K_2: 2\omega_s\)

Frequencies of four largest diurnal tides:
\(K_1: \omega_s\)
\(O_1: \omega_1 - \omega_2\)
\(P_1: \omega_0 - \omega_3\)
\(Q_1: \omega_1 - 2\omega_2 + \omega_4\)

Frequencies of two largest long-period tides:
\(M_f: 2\omega_2\)
\(M_m: \omega_2 - \omega_4\)
Tidal species

• Semidiurnal tides \((M_2, S_2, N_2, K_2)\):
  \[
  \eta_{EQ} = A(1 + k_2 - h_2)\cos^2(\phi)\cos(\omega t + 2\lambda),
  \]

• Diurnal tides \((K_1, O_1, P_1, Q_1)\):
  \[
  \eta_{EQ} = A(1 + k_2 - h_2)\sin(2\phi)\cos(\omega t + \lambda),
  \]

• Long-period tides \((M_f, M_m)\):
  \[
  \eta_{EQ} = A(1 + k_2 - h_2)\left[\frac{1}{2} - \frac{3}{2}\sin^2(\phi)\right]\cos(\omega t),
  \]

where \(\lambda\) is longitude wrt Greenwich, \(\phi\) is latitude, \(t\) is time wrt Greenwich, and \(A\) and \(\omega\) are constituent-dependent amplitudes and frequencies.

• \(h_2\): accounts for solid-earth body tide deformation

• \(k_2\): accounts for change in potential due to self-attraction of solid-earth deformation

• \((1 + k_2 - h_2) = 0.693\) for semidiurnal and long period tides, \(=\) up to \(0.736\) for diurnal tides due to “free-core nutation resonance” (Wahr 1981)
Self-attraction and loading

Earth yields to ocean loading. Potential altered by self-attractions of mass redistributions in earth and ocean (Hendershott 1972):

$$\eta_{SAL} = \sum_n \frac{3 \rho_{water}}{\rho_{earth}(2n+1)}(1 + k_n' - h_n') \eta_n$$

$\eta_n$ nth spherical harmonic of $\eta$

$k_n', h_n'$ load numbers (Munk and MacDonald 1960) from Farrell (1972)

Solve by iteration, starting with “scalar approximation” $\eta_{SAL} \approx 0.094 \eta$.

Attaining convergence in iteration non-trivial, use same trick that Egbert et al. (2004) used.
RMS $M_2$ elevation discrepancy between forward model with topographic wave drag and satellite-constrained model

(a)

(b)
Log_{10} M_2 dissipation in W m^{-2}; Optimal Topo Drag

Log_{10} M_2 dissipation in W m^{-2}; Optimal c_d
Elevation discrepancies (cm) of forward model, wrt to pelagic tide gauges

Without $S_2$ air tide forcing:

<table>
<thead>
<tr>
<th>Const</th>
<th>Signal</th>
<th>One-layer model</th>
<th>Two-layer model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_1$</td>
<td>1.62</td>
<td>0.43 (92.8)</td>
<td>0.40 (93.9)</td>
</tr>
<tr>
<td>$O_1$</td>
<td>7.76</td>
<td>1.79 (94.7)</td>
<td>1.62 (95.7)</td>
</tr>
<tr>
<td>$P_1$</td>
<td>3.62</td>
<td>0.84 (94.6)</td>
<td>0.73 (95.9)</td>
</tr>
<tr>
<td>$K_1$</td>
<td>11.26</td>
<td>2.90 (93.3)</td>
<td>2.26 (96.0)</td>
</tr>
<tr>
<td>$N_2$</td>
<td>6.86</td>
<td>1.95 (91.9)</td>
<td>1.83 (92.9)</td>
</tr>
<tr>
<td>$M_2$</td>
<td>33.22</td>
<td>9.33 (92.1)</td>
<td>8.75 (93.1)</td>
</tr>
<tr>
<td>$S_2$</td>
<td>12.66</td>
<td>4.40 (87.9)</td>
<td>4.21 (88.9)</td>
</tr>
<tr>
<td>$K_2$</td>
<td>3.43</td>
<td>1.00 (91.4)</td>
<td>0.96 (92.2)</td>
</tr>
<tr>
<td>RSS</td>
<td>39.06</td>
<td>11.12 (91.9)</td>
<td>10.34 (93.0)</td>
</tr>
</tbody>
</table>

With $S_2$ air tide forcing:

<table>
<thead>
<tr>
<th>Const</th>
<th>Signal</th>
<th>One-layer model</th>
<th>Two-layer model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_2$</td>
<td>12.66</td>
<td>3.29 (93.2)</td>
<td>3.10 (94.0)</td>
</tr>
<tr>
<td>RSS</td>
<td>39.06</td>
<td>10.73 (92.5)</td>
<td>9.94 (93.5)</td>
</tr>
</tbody>
</table>
Things to do in HYCOM

Run and test HYCOM as tide-only model.

Run tides and non-tidal motions simultaneously.

Topographic wave drag tricky, since it acts differently on tidal versus non-tidal motions (Bell 1975)

Use bottom boundary-layer drag and topographic wave drag to derive energy dissipation $\epsilon$, derive diffusivity $\kappa$ from microstructure formula $\kappa = \frac{\Gamma \epsilon}{N^2}$

Examine feedback of mixing onto large-scale circulation.